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Resource Depletion and Sustainability in Small Open Economies

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Exogenous price changes affect the amount that a small country exporting natural resource commodities must invest to sustain its consumption level. The necessary amount is given by the difference between Hotelling rent and the discounted sum of future terms-of-trade effects (capital gains). The latter term is found to be large relative to the former in the case of petroleum depletion in Indonesia. This suggests that resource-rich countries will need to invest more than previously expected to sustain their consumption levels, if natural resource prices continue their long-term historical decline. © 1997 Academic Press

I. INTRODUCTION

Hartwick [10], building upon Solow [23], identified the condition for sustaining consumption in a simple economy based on a nonrenewable natural resource:[1] investment in physical capital and other forms of reproducible capital must equal the economic depreciation of the resource. The latter, in turn, is given by Hotelling rent: the product of the quantity of resource extracted and the marginal rent (price minus marginal cost). In effect, diminutions in natural capital must be offset by increases in other forms of capital.[2] Dixit *et al.* [9] termed the sum of the values of changes in all forms of capital "net investment." Hence, consumption is sustainable only if net investment is nonnegative.

The models in Solow [23], Hartwick [10], and Dixit *et al.* [9] were for closed economies: economies in which prices of resources and other economic variables are generated endogenously. In practice, when one attempts to measure the sustainability of a particular country's economic course, one inevitably is dealing with an economy open to some amount of international trade and exhibiting some degree of price-taking behavior. Asheim [2, 3] and Hartwick [13] demonstrated that the basic net-investment rule requires reformulation in an open-economy context. What has remained unresolved, however, is the treatment of capital gains (positive or negative) arising from exogenous changes in the prices of extracted resources. This is of practical relevance as well as theoretical interest, as the prices of internationally traded natural resource commodities are notoriously volatile and

[1] We do not advocate constant consumption as a development strategy, but rather regard it as the minimum trend in consumption to which countries might aspire. Solow [23] argued for constant consumption as a definition of intergenerational equity.

[2] This is equivalent to Weitzman's [28] demonstration that net national product, properly defined, equals the stationary equivalent of long-term consumption possibilities along an optimal growth path. See also Mäler [16] and Dasgupta and Mäler [7].

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can rise or fall by significant amounts in a short period of time. The potential implications for long-run economic welfare in resource-exporting countries are especially important. Prices of oil and most other natural resources have declined substantially in real terms since the late 1800s.[3] Developing countries have long expressed concern about the effects of these declines upon their development prospects.

The literature on economic depreciation of natural resources contains only passing references to capital gains. The World Resources Institute (WRI; see Repetto *et al*. [20]) calculated ‘‘revaluation’’ terms for Indonesian petroleum and timber resources by multiplying resource stocks at the end of a period times the change in average rent (price of the extracted resource minus average extraction cost) during the period, but it did not include them in its economic depreciation estimates. Peskin [19] argued that economic depreciation should include capital gains, but he questioned the validity of calculating them by WRI's revaluation approach. Mäler [16] agreed with Peskin that economic depreciation should include capital gains, but only when price changes are unanticipated. Aslaksen *et al*. [4] stated that anticipated and unanticipated price changes affect the capitalized value of Norway's oil wealth, but they did not isolate capital gains from other factors affecting capitalized value, such as the extraction profile. Other than the WRI study of Indonesia, empirical applications of the net-investment rule, such as those by WRI for Costa Rica (see Repetto *et al*. [21]), van Tongeren *et al*. [26] for Mexico, Bartelmus *et al*. [5] for Papua New Guinea, Vincent [27] for Malaysia, and Pearce and Atkinson [18] and the World Bank [29] for a range of countries, have typically ignored the issue of capital gains.

In this paper, we resolve the capital gains issue in a theoretical context. We find that Peskin was correct in arguing that economic depreciation should include capital gains, even when price changes are anticipated, and that WRI's revaluation approach accurately measures capital gains only under extremely restrictive circumstances. We illustrate the correct calculation of capital gains by analyzing the economic depreciation of petroleum resources in Indonesia. The WRI study on this country [20] is probably the best-known empirical study on natural resource depletion and sustainability. We find that capital gains have a substantial impact on the estimated level of investment required to sustain Indonesia's consumption.

II. CONSTANT CONSUMPTION UNDER CONSTANT PRICES

This section presents our basic open-economy model and replicates key results from the closed-economy analyses of Hartwick [10] and Dixit *et al*. [9]. The model is for a nonrenewable resource, which for convenience we label ‘‘oil.’’ The next section contains our new findings related to capital gains.

Consider an oil republic (OR) specializing in oil extraction and export. Its population is constant, and its consumption goods are all imported at a constant price. Its oil is homogeneous and is sold at world prices, which are exogenous to the OR. Oil extraction is costly, with an upward-sloping marginal cost curve (marginal cost rises with the amount extracted). The marginal cost curve does not shift over time: there is no technical progress or investment in extraction capital,

[3] There is a large literature on this subject. For a recent review, see Nordhaus [17].

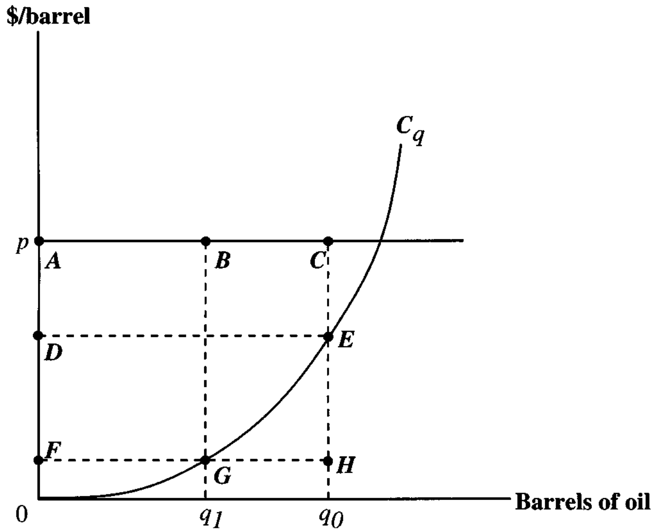

FIG. 1. Constant consumption under constant prices.

and there are no stock effects. Oil owners invest part of the net return (resource rent) from current oil production abroad, at a constant interest rate r. Hence, current consumption is financed out of noninvested oil rents and interest payments from past foreign investments.

Figure 1 illustrates the standard net-investment rule for the first two periods of an optimal extraction program when the world price of oil is constant at p. Current oil production is given by q, and the marginal cost curve for oil extraction is C_q.[4] By Hotelling's Rule, marginal rent in period 1 (distance BG) is $1 + r$ times marginal rent in period 0 (distance CE). Oil revenue in each period divides naturally into total extraction costs ($C(q)$, the area under the marginal cost curve), inframarginal rent (marginal cost times production, minus total extraction costs), and Hotelling rent. Since period 0 is the initial period, consumption in this period simply equals the inframarginal rent "triangle $0DE$. If the Hotelling rent in period 0, the rectangle $DACE$, is invested abroad, then consumption in period 1 equals the smaller triangle $0FG$, plus the return on the investment. Since $BG/CE = 1 + r$, EH equals rCE, and so this return is $FDEH$. The "triangle" GEH is vanishingly small in continuous time, and so consumption in period 0 equals consumption in period 1.[5]

More formally, let $K(t)$ be accumulated savings invested abroad as of period t and earning interest rate r. Then aggregate consumption can be written as

$$Z(t) = pq(t) - C(q(t)) - \dot{K}(t) + rK(t). \tag{1}$$

[4] Here and elsewhere, we ignore the effects of stock changes, discoveries, and technological improvements on the costs of oil extraction.

[5] In discrete time, the triangle GEH does not disappear (Dasgupta and Mitra [8]; Long *et al.* [15]). Consequently, consumption in period 1 will be higher than in period 0. To keep consumption constant, less than the Hotelling rent should be invested. Hartwick [12] recently demonstrated that the appropriate amount to invest is given by the product of current production and marginal rent from the *previous* period.

Here and elsewhere, a dot above a variable or an expression indicates a time derivative. If (i) investment is set equal to the Hotelling rent for current extraction,

$$\dot{K}(t) = \left[p - C_q(q(t))\right]q(t), \tag{2}$$

and (ii) oil is extracted in a profit-maximizing fashion (hence, the Hotelling r-percent rule holds),

$$\frac{\dot{\overline{p - C_q(q(t))}}}{p - C_q(q(t))} = r, \tag{3}$$

then we can readily show that $\dot{Z}(t)$ equals zero. Hence, consumption is constant at $\overline{Z}$.[6] Appendix 1 contains the details.

What is the level of $\overline{Z}$? Note that at any point along the constant-consumption path, the OR could in principle auction off its remaining stock of oil $S(t)$ and, assuming competitive bidding, receive the capitalized value

$$V(S(t)) = \int_t^T e^{-r(\tau - t)}\left[pq^*(\tau) - C(q^*(\tau))\right] d\tau. \tag{4}$$

The asterisks (*) indicate rent-maximizing quantities. Consumption in remaining periods would then equal the interest on the invested auction proceeds, plus the return on investments made in earlier periods. This implies that $\overline{Z}$ can be viewed as interest on a constant total capital stock equal to the sum of foreign investments and the capitalized value of oil resources. That is, $\overline{Z} = r[K(t) + V(S(t))]$ (see Appendix 2 and Solow [24]).

III. CONSTANT CONSUMPTION UNDER CHANGING PRICES

A changing oil price, with its future path known to the OR (due, for example, to the existence of a complete set of futures markets) and therefore fully anticipated, introduces nontrivial price effects into the model.[7] Suppose that the price is falling, although not necessarily at a constant rate. Now, if the OR invests Hotelling rents abroad and lives off interest and inframarginal rents, its consumption will fall instead of remaining constant. Figure 2 illustrates this. The price falls from p_0 to p_1 between period 0 and period 1. The new optimal production levels, q_0 and q_1, are higher and lower, respectively, than the corresponding values in Fig. 1, as production shifts toward the present, when price is higher. These adjustments maintain the relationship that marginal rent in period 1 (the distance LR) equals $1 + r$ times marginal rent in period 0 (the distance JP). If the Hotelling rent in

[6] A rising path of consumption would require investment of more than the Hotelling rent. Because the discount rate and the rate of return on investment are the same in our simple model, the discounted sum of consumption would be the same along constant-consumption and rising-consumption (and for that matter, declining-consumption) paths. That is, the constant-consumption path is unique, but it is not the only efficient consumption path.

[7] Oil discoveries that occur in the rest of the world but not in the OR offer one explanation for exogenous price changes. Another is improvements in extraction technology that are adopted only in the rest of the world.

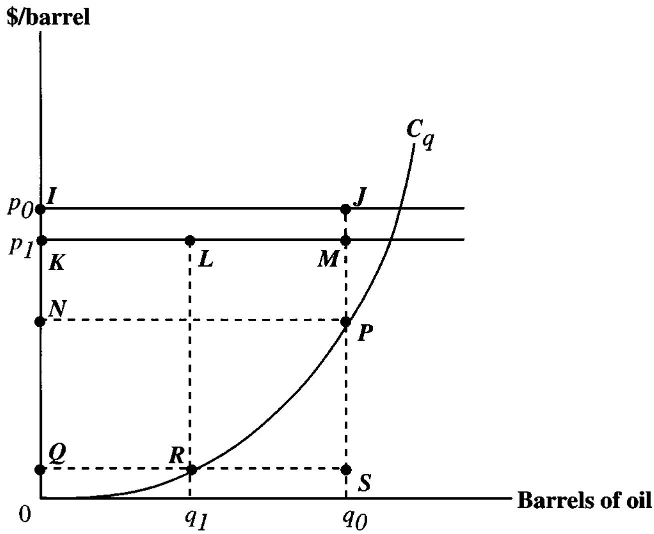

FIG. 2. Constant consumption under rising prices.

period 0 is invested abroad, then consumption in period 1 will be *0QR* (inframarginal rent) plus *QNPS*—*KIJM* (return on investment, as *LR*−*JP* = *NQ*−*IK*). In continuous time, this equals *0NP* minus *KIJM*. But *0NP* is consumption in period 0. Hence, consumption is lower in period 1, with the amount of the reduction related to the magnitude of the price decline (the distance *IK*).

In short, the price decline in period 1 offsets some of the investment return on the Hotelling rent. The OR must therefore consume less and invest more in period 0 to maintain consumption constant. It must invest not only the Hotelling rent but some of the inframarginal rent as well.

More formally, let the oil price $p(t)$ be some function of time.[8] This function is not necessarily monotonic. Consumption is now given by

$$Z(t) = p(t)q(t) - C(q(t)) - \dot{K}(t) + rK(t). \tag{5}$$

The investment rule and extraction-efficiency condition corresponding to (2) and (3) are

$$\dot{K}(t) = \left[p(t) - C_q q(t)\right]q(t) \tag{6}$$

and

$$\frac{\dot{\overline{p(t) - C_q(q(t))}}}{p(t) - C_q(q(t))} = r. \tag{7}$$

Derivations analogous to those in Appendix 1 can be used to show that, under (6) and (7), consumption is changing by $\dot{p}(t)q(t)$ each period (*KIJM* in Fig. 2). Asheim

[8] Martin Weitzman, in a comment at a seminar, pointed out that exogenously changing prices are just one type of nonautonomous economic change relevant to the issue of sustainable development. Perhaps more important is technological progress.

[2] referred to this quantity as a terms-of-trade shift. In Appendix 3, we show that the level of investment that maintains consumption at a constant level is

$$\dot{K} = \left[p(t) - C_q(q(t)) \right] q(t) - \int_t^T e^{-r(\tau - t)} \dot{p}(\tau) q^*(\tau)\, d\tau. \tag{8}$$

The integral on the right-hand side gives the present value of future terms-of-trade shifts, i.e., the capital gain (possibly negative) on the resource stock.[9] It goes to zero when $\dot{p}(\tau) = 0 \forall \tau$, causing (8) to collapse to (6). Note that the integral equals revaluation as defined by WRI (change in average rent times resource stock) only if the following extremely restrictive conditions hold simultaneously: the discount rate equals zero, the price change is constant over time, and the price change equals the change in average rent.

We noted in the previous section that aggregate wealth remains constant along the constant-consumption path when oil prices are constant. This is also true under the investment strategy given by (8). Aggregate wealth is now

$$W(t) = K(t) + V(S(t), t), \tag{9}$$

where

$$V(S(t), t) = \int_t^T e^{-r(\tau - t)} \left[p(\tau) q^*(\tau) - C(q^*(\tau)) \right] d\tau. \tag{10}$$

The change in aggregate wealth is

$$\dot{W}(t) = \dot{K}(t) + rV(S(t), t) - \left[p(t) q^*(t) - C(q^*(t)) \right]. \tag{11}$$

When $\dot{K}(t)$ is determined by (8), it can be shown that $\dot{W}(t) = 0$ (see Appendix 4).

IV. AN ILLUSTRATION: PETROLEUM DEPLETION IN INDONESIA

We examined the potential empirical significance of the capital gains term by analyzing petroleum depletion in Indonesia. The WRI study covered the period 1971–1984 and included three resources, petroleum, timber, and agricultural soils. In calculating economic depreciation allowances for these resources, WRI used average costs instead of marginal costs. That is, it set the allowances equal to total rent, not Hotelling rent.[10] From Fig. 1, it is evident that this procedure overstates economic depreciation when prices are constant. WRI's total rent estimates for petroleum are shown in the first column of the "Actual" section of Table 1.[11]

[9] In discrete time, there is a "triangle" in Fig. 2 (RPS) that no longer vanishes, and this causes the investment rule given by (8) to fail to maintain consumption at constant levels. It appears that an empirically applicable, discrete-time analogue to (8) can be derived only by placing restrictions on the time path of $p(t)$ (e.g., a constant rate of change). In this paper, we content ourselves with the derivation of (8), while bearing in mind that application of this formula to discrete-time data, as in the Indonesia example, generates inexact estimates of the amount of investment necessary to keep consumption constant.

[10] All other empirical studies cited in the introduction also used total rent, except Vincent [27].

[11] Annual exchange rates for converting the price and cost data in Table II.4 of the WRI report from nominal U.S. dollars to real rupiah were calculated by dividing values in the "Net Change" row of the "Monetary Accounts" section of that table by corresponding values in the "Petroleum" column in Table I.2. The overall depreciation allowance calculated by WRI included changes in oil stocks due to not only oil production but also discoveries and revisions in reserves. We have ignored the latter adjustments here, as it is not clear that they should be included in an economic measure of resource depletion (Bartelmus *et al.* [6], Hartwick and Hageman [14]).

TABLE 1
Rent, Hotelling Rent, and Economic Depreciation for Petroleum in Indonesia (billion 1973 rupiah)

I. Actual				II. Projections			
Year	Total rent[a]	Hotelling rent[b]	Economic depreciation[c]	Year	Total rent[a]	Hotelling rent[b]	Economic depreciation[c]
1971	274	−163	−1068	1985	2540	287	578
1972	477	−12	−901	1986	2487	320	598
1973	594	129	−914	1987	2435	356	620
1974	1286	579	161	1988	2383	396	646
1975	1223	416	−6	1989	2332	440	676
1976	1270	533	−150	1990	2281	488	709
1977	1452	941	96	1991	2230	540	747
1978	1344	861	−159	1992	2178	598	788
1979	1383	737	−274	1993	2125	659	834
1980	2161	1193	957	1994	2071	725	883
1981	2505	1204	1360	1995	2014	795	935
1982	1796	168	264	1996	1953	868	990
1983	2113	−50	380	1997	1886	942	1046
1984	2028	201	505	1998	1810	1014	1099
				1999	1721	1079	1145
				2000	1612	1128	1175
				2001	1468	1144	1172
				2002	1257	1088	1100
				2003	855	820	820

[a] Output times average rent.
[b] Output times marginal rent (right-hand side of (6)).
[c] Hotelling rent, minus capital gain (right-hand side of (8)).

These values exceeded the combined value of the depreciation allowances for the other two resources in nearly every year, usually by a substantial amount. They were cumulatively equivalent to over two-thirds of total gross investment in the economy during 1971–1984.[12] The impacts on net investment are not clear, however, not only because total rent exceeds economic depreciation for nonrenewable natural resources but also because data limitations prevented WRI from estimating the economic depreciation of other forms of capital (e.g., equipment and structures).

Estimating Hotelling rents requires estimates of marginal costs. But as Adelman and Shahi [1] noted, reliable, well-documented cost data are difficult to obtain for oil-exporting countries. Indonesia is no exception. In the absence of direct data, we constructed marginal cost estimates by assuming that the marginal cost curve for petroleum was isoelastic: $C_q = \alpha q^{\beta}$. Under this assumption, marginal cost equaled the WRI average cost estimate multiplied by $1 + \beta$.

We selected the elasticity to be consistent with the optimal depletion of Indonesia's petroleum reserves in the period beyond the end of the WRI sample, i.e., after

[12] Based on values in Table I.2 in the WRI report.

1984.[13] To model optimal depletion, we first projected oil prices beyond 1984 by applying an annual rate of price change of -1.5% per year, the long-run trend in oil prices during the past century [17].[14] Next, we calibrated the marginal cost curve to the means of the 1971–1984 data to derive the value of α corresponding to a given value of β. Then, we projected marginal rents forward from 1984 by applying Hotelling's rule. In doing so, we assumed a discount rate of 12.5%, which is the mean of the range of rates used by the Indonesian Ministry of Finance.[15] Subtracting the projected marginal rent series from the projected price series yielded projections of marginal cost. In turn, this series yielded projections of optimal production levels when inserted into the inverse of the marginal cost curve. We started with an initial elasticity estimate of $\beta = 1$ and increased it until we obtained a value that resulted in the complete exhaustion of oil reserves at the point when production went to zero. This value was 2.865, and the projected final year of production (T) was 2003.

The second column of the "Actual" section of Table 1 shows the 1971–1984 estimates of Hotelling rent (i.e., the right-hand side of (6)) at this elasticity value.[16] All of the estimates are much smaller than the total rent estimates, suggesting that Indonesia's prospects for sustainability were much better than WRI concluded. Does this remain true after taking changes in oil prices into account? The projections of price changes and production levels through 2003 enabled us to calculate the integral in (8). The third column of the "Actual" section of Table 1 shows the resulting estimates of the right-hand side of (8) for 1971–1984. Although real oil prices have declined in the long run, they rose substantially during 1971–1984 due to the first and second oil shocks. Not surprisingly, most estimates in the third column, which incorporate the capital gains associated with those price rises, are much smaller than the corresponding Hotelling rent estimates in the second column. Thanks to the rising prices, Indonesia did not need to invest even as much as the Hotelling rent in most years during 1971–1984 to offset the economic depreciation of its petroleum resources.

Because we projected declining prices during 1985–2003, the opposite is true for the post-1984 period (see the "Projections" section of Table 1): Indonesia has needed to invest substantially more than the Hotelling rent since the end of the oil-shock era. The required amount of investment was more than double the Hotelling rent in the second half of the 1980s. It is projected to decline toward the

[13] We used this implicit procedure for determining the elasticity because econometric estimation of the elasticity based on the WRI data did not yield satisfactory results. This is not surprising, given the short time series and the very approximate nature of the cost data.

[14] This procedure understates the actual price decline that occurred during the first decade of the projection period: according to data in the IMF's 1995 *International Financial Statistics Yearbook*, the crude petroleum export unit value in Indonesia (real, in rupiah) declined at an average annual rate of 4.9% during 1984–1993. Most of the decline occurred in the first few years of the period, however, suggesting that this rate should not be used for the entire projection period.

[15] Personal communication from Michael Roemer, Harvard Institute for International Development. Such a high social discount rate is plausible in a fast-growing economy like Indonesia's, due to the high return to capital investments (which causes the social opportunity cost of capital to be high) and the high income growth rate (which causes the social rate of time preference to be high).

[16] The negative values in a few years result from the marginal cost estimates slightly exceeding the petroleum price. This implies that actual petroleum production levels were not optimal. Over most of the period, however, the trajectory of estimated marginal rents was consistent with a double-digit discount rate, as was assumed.

Hotelling rent as the estimated depletion date approaches, because the integral includes fewer and fewer remaining years of production. In no period, however, is it as large as the total rent.[17]

V. CONCLUSIONS

Theoretical considerations imply that net investment calculations for resource-exporting nations must take into account not only Hotelling rent but also the capital gains resulting from exogenous price changes, even when the price changes are anticipated. The capital-gains adjustment is not, however, as simple as the product of price change and resource stock. The example of petroleum depletion in Indonesia demonstrates that the adjustment is empirically feasible and can have a substantial impact on the estimated amount of investment required to offset the economic depreciation of natural resources.

Rising oil prices during 1971–1984 apparently reduced the required amount of investment in Indonesia to less than Hotelling rent, but falling prices since (and projections of continued declines into the future) have apparently caused it to exceed Hotelling rent. Overly optimistic projections of resource prices perhaps help explain why resource-rich countries have, on average, grown less rapidly than resource-poor countries [22]. Mexico, Nigeria, and Venezuela are well-known cases of oil exporters that overconsumed and underinvested during the boom years of the 1970s and consequently encountered severe debt problems once the boom ended. Indonesia maintained high savings and investment rates during that period and thus avoided this fate. Application of the revised net-investment rule developed in this paper might have helped macroeconomic policymakers guide less well-managed resource-exporting countries toward more sustainable development paths.

Of course, the accuracy of the price forecasts employed by policymakers is of critical importance. As it turned out, many oil analysts in the 1970s overestimated the strength of OPEC and failed to predict the mid-1980s return of oil prices to their long-run downward trend. To avoid being misled by cyclical fluctuations and dramatic, but transient, price shocks such as those that occurred in the 1970s, it is probably best to use long-run trends in applying the revised net-investment rule. The century-old decline in most resource prices suggests that underinvestment, not overinvestment, is the principal risk facing resource exporters.

APPENDIX 1: CONSTANT CONSUMPTION WITH CONSTANT RESOURCE PRICE

The OR's economy comprises: (i) the accounting relation given by (1), (ii) the savings–investment relation given by (2), and (iii) the efficiency condition given by (3). The time derivatives of (1) and (2) are

$$\dot{Z} = \left[p - C_q(q(t)) \right] \dot{q} - \ddot{K} + r\dot{K} \qquad \text{(A1)}$$

[17] Except at the moment of depletion. In Table 1, economic depreciation does not quite equal total rent in the year 2003 due to discrete-time approximation errors.

and

$$\ddot{K} = \left[p - C_q(q(t))\right]\dot{q} + \dot{\overline{\left[p - C_q(q(t))\right]}}q(t). \tag{A2}$$

If we substitute: (i) $r[p - C_q(q(t))]$ from (3) for $\dot{\overline{p - C_q(q(t))}}$ in (A2), (ii) the resulting modified version of (A2) for $\ddot{K}$ in (A1), and (iii) (2) for $\dot{K}$ in (A1), we obtain the result that $\dot{Z} = 0$ (consumption is constant).

APPENDIX 2: CONSTANT WEALTH WITH CONSTANT RESOURCE PRICE

At any date, the OR's wealth comprises $K(t)$ dollars invested abroad plus the capital value of its remaining oil stock, $V(S(t))$:

$$W(t) = K(t) + V(S(t)) \tag{A3}$$

Substituting (4) for $V(S(t))$ and taking the time derivative, we obtain

$$\dot{W}(t) = \dot{K}(t) + rV(S(t)) - \left[pq^*(t) - C(q^*(t))\right]. \tag{A4}$$

In taking the derivative of $V(S(t))$, the derivative with respect to terminal date T vanishes due to an envelope condition (the end-point optimal condition). From dynamic programming, we know that for an optimal extraction program,

$$rV(S(t)) = pq^*(t) - C(q^*(t)) - \left[p - C_q(q^*(t))\right]q^*(t). \tag{A5}$$

If we substitute this into (A4), we obtain

$$\dot{W}(t) = \dot{K}(t) - \left[p - C_q(q^*(t))\right]q^*(t). \tag{A6}$$

According to (2), investment is being financed from current Hotelling rents. Hence, the right-hand side of (A6) is zero, and wealth remains constant.

APPENDIX 3: CONSTANT CONSUMPTION WITH CHANGING RESOURCE PRICE

The time derivatives of (5) and (8) are

$$\dot{Z}(t) = \left[p(t) - C_q(q(t))\right]\dot{q}(t) + q(t)\dot{p}(t) - \ddot{K}(t) + r\dot{K}(t) \tag{A7}$$

and

$$\ddot{K} = \left[p(t) - C_q(q(t))\right]\dot{q}(t) + q(t)\dot{\overline{\left[p(t) - C_q(q(t))\right]}} - r\int_t^T e^{-r(\tau - t)}\dot{p}(\tau)p(\tau)\,d\tau + \dot{p}(t)q(t). \tag{A8}$$

Substitute (A8) for $\ddot{K}$ in (A7) to obtain

$$\dot{Z}(t) = -q(t)\dot{\overline{\left[p(t) - C_q(q(t))\right]}} + r\int_t^T e^{-r(\tau - t)}\dot{p}(\tau)q(t)\,d\tau + r\dot{K}(t). \tag{A9}$$

From (7), dynamic efficiency implies

$$\dot{\overline{p(t) - C_q(q(t))}} = r\left[p(t) - C_q(q(t))\right]. \quad \text{(A10)}$$

Hence,

$$\dot{Z}(t) = r\int_t^T e^{-r(\tau - t)}\dot{p}(\tau)q(\tau)\,d\tau - rq(t)\left[p(t) - C_q(q(t))\right] + r\dot{K}(t). \quad \text{(A11)}$$

The right-hand side is zero (consumption is constant) if the investment rule given by (8) is followed.

APPENDIX 4: CONSTANT WEALTH WITH CHANGING RESOURCE PRICE

The change in the value of the remaining resource stock (10) over time is

$$\dot{V}(s(t),t) = rV(S(t),t) - \left[p(t)q^*(t) - C(q^*(t))\right]. \quad \text{(A12)}$$

If we define

$$J(S(t),t) = e^{-rt}V(S(t),t), \quad \text{(A13)}$$

then from dynamic programming we have

$$-J_t = e^{-rt}\left[p(t)q^*(t) - C(q^*(t))\right] - J_S q^*(t). \quad \text{(A14)}$$

The partial derivatives in this expression are

$$J_t = -re^{-rt}V(S(t),t) + e^{-rt}\,\partial V/\partial t \quad \text{(A15)}$$

and

$$J_S = e^{-rt}V_S(S(t),t), \quad \text{(A16)}$$

where

$$V_S = \left[p(t) - C_q(q^*(t))\right] \quad \text{(A17)}$$

and

$$\frac{\partial V}{\partial t} = \int_t^T \dot{p}(\tau)q^*(\tau)\,e^{-r(\tau - t)}\,dt \quad \text{(A18)}$$

After substituting (A15) for J_t and (A16) for J_S, (A14) becomes

$$rV(S(t),t) = p(t)q^*(t) - C(q^*(t)) - V_S q^*(t) + \partial V/\partial t.$$

This, along with (A17) and (A18), can be substituted into (A12) to yield the economic depreciation expression,

$$\dot{V}(S(t),t) = -\left[p(t) - C_q(q^*(t))\right]q^*(t) + \int_t^T e^{-r(\tau - t)}\dot{p}(\tau)q^*(\tau)\,d\tau. \quad \text{(A19)}$$

Substituting this into the time derivative of the aggregate wealth expression given by (9), we obtain

$$\dot{W}(t) = \dot{K}(t) - \left[p(t) - C_q(q^*(t))\right]q^*(t) + \int_t^T e^{-r(\tau - t)}\dot{p}(\tau)q^*(\tau)\,d\tau.$$

This equals zero (wealth is constant) if the investment rule given by (8) is followed.

APPENDIX 5: NOMENCLATURE

C	Cost of oil extraction
C_q	Marginal cost of oil extraction
e	Mathematical constant (= 2.718...)
J	Value of stock of remaining oil, constant terms (year 0)
K	Savings invested abroad
p	Price of extracted oil
q	Oil output
r	Interest rate
S	Stock of remaining oil
t	Time period
T	Period when oil is exhausted
V	Value of stock of remaining oil, current terms (year t)
W	Aggregate wealth
Z	Aggregate consumption
α	Constant in marginal cost curve
β	Exponent in marginal cost curve
τ	Dummy of integration

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