Homework | Answer Key

1. At any given set of prices, the excess demands equal excess supply. In addition, the monetary value of goods a consumer plans to purchase in any period must be equal to the monetary value of goods/services the consumer plans to sell in that period.

Significant result: If N-1 markets clear then the N^{th} market also clears.

Walras law is especially important in GE models because

1) allows us to find a price vector to ensure demand = supply in all mrkts
2) If scarcity exists in economy then all income will be spent.
   ↳ This allows us to use equality on budget constraints
   $Y = P_x X$.

- Answers will vary slightly.

2. $U_T = G_T H_T$

$U_M = G_M H_M^2$

$G_T + G_M = 75$

$H_T + H_M = 100$

a. To solve for the contract curve, we need to remember that the contract curve makes up all of the P.O. points of trade. This requires that

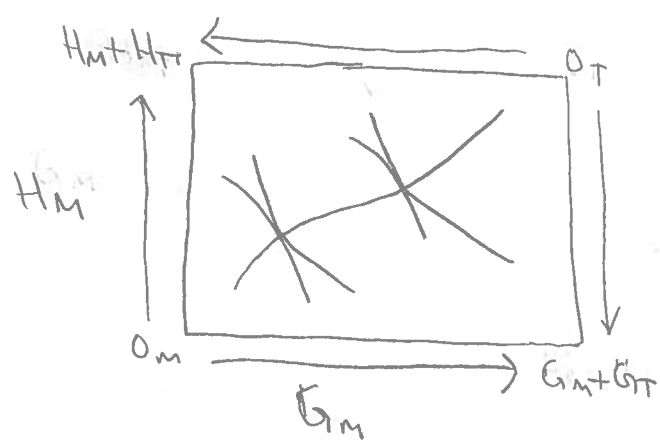
1) $MRS_T = MRS_M$
2) $75 = G_T + G_M$
3) $100 = H_T + H_M$

Let G exist on the ~~x~~-axis and H on the y-axis.

$$MRS_T = -\frac{MU_G}{MU_H} = -\frac{H_T}{G_T}$$

$$MRS_M = -\frac{MU_G}{MU_H} = \frac{-H_M^2}{2G_M H_M} = -\frac{H_M}{2G_M}$$

$$MRS_T = MRS_M$$



$$\frac{H_T}{G_T} = \frac{H_M}{2G_M}$$

$$H_M = \frac{2G_M H_T}{G_T}$$

From (2) (3)

$$G_T = 75 - G_M$$
$$H_T = 100 - H_M$$

$$H_M = \frac{2G_M(100 - H_M)}{(75 - G_M)} ; = H_M 75 - G_M H_M = 200 G_M - 2G_M H_M$$
$$; = H_M 75 + G_M H_M = 200 G_M$$

$$H_M(75 + G_M) = 200 G_M$$

$$\boxed{H_M = \frac{200 G_M}{75 + G_M}}$$

b.

MRS$_M$ = price ratio

$$\frac{H_M}{2G_M} = P$$

$$y_M = PG_M + H_M$$

$$H_M = P2G_M$$

$$y_M = PG_M + P2G_M$$

$$y_M = 3PG_M$$

$$G_M = \frac{y_M}{3P} \quad ; \quad H_M = \frac{2}{3}y_M$$

MRS$_T$ = price ratio

$$\frac{H_T}{G_T} = P \qquad y_T = PG_T + H_T$$

$$H_T = G_T P$$

$$y_T = PG_T + PG_T = 2PG_T$$

$$G_T = \frac{y_T}{2P} \quad ; \quad H_T = \frac{y_T}{2}$$

$$G_T + G_M = 100$$

$$\underbrace{\frac{y_T}{2P} + \frac{y_M}{3P}}_{\text{Sum of demand}} = \underbrace{100}_{\text{endowment (supply)}}$$

$$\frac{3y_T + 2y_m}{6P} = 100$$

$$\frac{3y_T + 2y_m}{600} = P$$

$$\frac{3(PG_T + H_T) + 2(PG_M + H_M)}{600} = P$$

$$\frac{3PG_T}{600} + \frac{3H_T}{600} + \frac{2PG_M}{600} + \frac{2H_M}{600} = P$$

$$\frac{3H_T + 2H_M}{600} = P - \frac{3PG_T}{600} - \frac{2PG_M}{600}$$

$$\frac{3H_T + 2H_M}{\cancel{600}} = \frac{600P - 3PG_T - 2PG_M}{\cancel{600}} = (600 - 3G_T - 2G_M)P$$

$$\boxed{P = \frac{3H_T + 2H_M}{600 - 3G_T - 2G_M}}$$

c. See part a.

d. Yes, it would be a straight line unless one good is an inferior good and endowment/income differs for each person.

5. a <u>centralized</u>

$$\max_{g_A, b_A, g_B, b_A, g, b, x_g, x_b} U_A(g_A, b_A) = g_A^{0.4} b_A^{0.6}$$

s.t.

$$\bar{U}_B \leq 10 + 0.7\ln(g_B) + 0.3\ln(b_B)$$

$$g = x_g$$

$$b = 2x_b$$

$$g = g_A + g_b$$

$$b = b_A + b_B$$

$$\bar{x} = x_A + x_B = x_g + x_b$$

$$L(\cdot) = g_A^{0.4} b_A^{0.6} + \mu(10 + .7\ln(g_B) + .3\ln(b_B)) +$$
$$\lambda^g (x_g - g) +$$
$$\lambda^b (2x_b - b) +$$
$$\gamma^g (g - g_A - g_b) +$$
$$\gamma^b (b - b_A - b_B) +$$
$$\gamma^x (\bar{x} - x_g - x_b)$$

<u>FOC's</u>

$$L_{g_A} = 0.4 g_A^{-0.6} b_A^{0.6} - \gamma^g = 0 \quad 1.$$

$$L_{b_A} = 0.6 b_a^{-0.4} g_A^{0.6} - \gamma^b = 0 \quad 2.$$

$$L_{g_B} = \mu .7 g_B^{-1} - \gamma^g = 0 \quad 3.$$

$$L_{b_B} = \mu .3 b_B^{-1} - \gamma^b = 0 \quad 4.$$

$$L_g = -\lambda^g + \gamma^g = 0 \quad 5.$$

$$L_b = -\lambda^b + \gamma^b = 0 \quad 6.$$

$$L_{x_g} = \lambda^g - \gamma^x = 0 \quad 7.$$

$$L_{x_b} = 2\lambda^b - \gamma^x = 0 \quad 8$$

$$\frac{(1)}{(2)} = \frac{0.4 g_A^{-0.6} b_A^{0.6}}{0.6 b_A^{-0.4} g_A^{0.6}} = \frac{\gamma^g}{\gamma^b} \quad (9)$$

$$= \underbrace{\frac{2 b_A}{3 g_A}}_{MRS_A} = \underbrace{\frac{\gamma^g}{\gamma^b}}_{\text{price ratio (shadow)}} \quad (10)$$

$$\frac{(3)}{(4)} = \frac{\cancel{1}.7 g_B^{-1}}{\cancel{1}.3 b_B^{-1}} = \frac{\gamma^g}{\gamma^b} \quad (11)$$

$$\underbrace{\frac{7 b_B}{3 g_B}}_{MRS_B} = \underbrace{\frac{\gamma^g}{\gamma^b}}_{\text{price ratio (shadow)}} \quad (112)$$

$$\frac{(5)}{(6)} = \underbrace{\frac{\lambda^g}{\lambda^b}}_{\text{price ratio}} = \frac{\gamma^g}{\gamma^b} \quad (13)$$

$$\frac{(7)}{(8)} = \frac{\lambda^g}{2\lambda^b} = 1 \quad (14)$$

$$= \frac{\lambda^g}{\lambda^b} = \underbrace{2}_{MRT}$$

from (13)

$$= \underbrace{\frac{\gamma^g}{\gamma^b}}_{\text{priceratio}} = \underbrace{2}_{MRT}$$

$\therefore$

$$\underbrace{2\frac{b_A}{g_A}}_{MRS_A} = \underbrace{\frac{7b_B}{3g_B}}_{MRS_B} = \underbrace{2}_{MRT} = \underbrace{\frac{\gamma^g}{\gamma^b}}_{\text{price ratio}}$$

> providing the efficient allocation of inputs across outputs!

b. Decentralized or competitive eq.

Consumers

Person A

$$\max_{g_A, b_A} \quad u^A(g_A, b_A) = g_A^{0.4} b_A^{0.6}$$

$$\text{s.t.} \quad y_A = p_g g_A + p_b b_A$$

$$L(.) = g_A^{0.4} b_A^{0.6} + \lambda (y_A - p_g g_A - p_b b_A)$$

FOC

$$L_{g_A} = 0.4 g_A^{-0.6} b_A^{0.6} - \lambda p_g = 0 \qquad (1a)$$

$$L_{b_A} = 0.6 g_A^{0.4} b_A^{-0.4} - \lambda p_b = 0 \qquad (2a)$$

$$\frac{(1a)}{(2a)} = \underbrace{\frac{2 b_A}{3 g_A}}_{MRS_A} = \underbrace{\frac{p_g}{p_b}}_{\text{price ratio}}$$

Person B

$$\max_{g_B, b_B} \quad u^B(g_B, b_B) = 10 + 0.7 \ln(g_B) + 0.3 \ln(b_B)$$

$$\text{s.t.} \quad y_B = p_g g_B + p_b b_B$$

$$L(.) = 10 + .7 \ln(g_B) + 0.3 \ln(b_B) + \lambda (y_B - p_g g_B - p_b b_B)$$

FOC

$$L_{g_B} = .7 g_B^{-1} - \lambda p_g = 0 \qquad (3a)$$

$$L_{b_B} = .3 b_B^{-1} - \lambda p_b = 0 \qquad (4a)$$

$$\frac{(3a)}{(4a)} = \underbrace{\frac{7 b_B}{3 g_B}}_{MRS_B} = \underbrace{\frac{p_g}{p_b}}_{\text{price ratio}}$$

Firms

Firm g. (Almonds)

$$\max_{x_g} \; p_g x_g - x_g$$

FOC

$$L_{x_g} = p_g - 1 = 0$$

$$p_g = 1 \qquad (5a)$$

Firm b (grapes)

$$\max_{x_b} \; p_b 2x_b - x_b$$

FOC

$$L_{x_b} = p_b 2 - 1 = 0$$

$$2p_b = 1 \qquad (6a)$$

$$\left(\frac{5a}{6a}\right) = \frac{p_g}{2p_b} = 1$$

$$= \frac{p_g}{p_b} = \underbrace{2}_{MRT}$$

price ratio = MRT

$\therefore$

$$\underbrace{\frac{2b_A}{3g_A}}_{MRS_A} = \frac{7b_B}{3g_B} = 2 = \frac{p_g}{p_b}$$

$MRS_A = MRS_B = MRT = $ price ratio

$\therefore$ Decentralized = Centralized Solution!

c. Letting consumers income = 20 (endowment) $= X_i(1)$ 9.

Find each person's demand for g & b.

A

$$20 - P_g g_A - \cancel{P_b}\left(\frac{P_g}{\cancel{P_b}}\frac{3}{2}g_A\right) = 0$$

$$P_g g_A + \frac{3}{2}P_g g_A = 20$$

$$g_A = \frac{20}{(1+\frac{3}{2})P_g} = \frac{20}{2.5P_g} \boxed{= \frac{8}{P_g}}$$

$$20 - \cancel{P_g}\left(\frac{8}{\cancel{P_g}}\right) - P_b b_A = 0$$

$$P_b b_A = 12$$

$$\boxed{b_A = \frac{12}{P_b}}$$

B.

$$20 - P_g g_B - \cancel{P_b}\left(\frac{P_g}{\cancel{P_b}}\frac{3}{7}g_B\right) = 0$$

$$20 - P_g g_B - \frac{3}{7}P_g g_B$$

$$g_B = \frac{20}{(1+\frac{3}{7})P_g} = \frac{20}{(\frac{10}{7})P_g} \boxed{= \frac{14}{P_g}}$$

$$20 - \cancel{P_g}\left(\frac{14}{\cancel{P_g}}\right) - P_b b_B = 0$$

$$6 - P_b b_B = 0$$

$$\boxed{b_B = \frac{6}{P_B}}$$

Markets

Almonds

$$\hat{g}_A + \hat{g}_B = g = X_g$$

$$\frac{8}{P_g} + \frac{14}{P_g} = X_g$$

from firm foc, $P_g = 1$, $P_b = \frac{1}{2}$

$$\frac{8}{1} + \frac{14}{1} = X_g$$

$$\boxed{X_g = 22}$$

Grapes

$$\hat{b}_A + \hat{b}_B = b = 2X_b$$

$$\frac{12}{P_b} + \frac{6}{P_b} = 2X_b$$

$$\frac{12}{1/2} + \frac{6}{1/2} = 2X_b$$

$$24 + 12 = 2X_b$$

$$\boxed{X_b = 18}$$

$$\boxed{\begin{matrix} P_g = 1 & x = 1 \\ P_b = \frac{1}{2} & \end{matrix}}$$

d. $X_g = 22$ $\qquad X_g + X_b = \underline{40!}$

$X_b = 18$

$g_A = 8$ $\qquad g_A + g_B = g = X_g = 22!$

$g_B = 14$

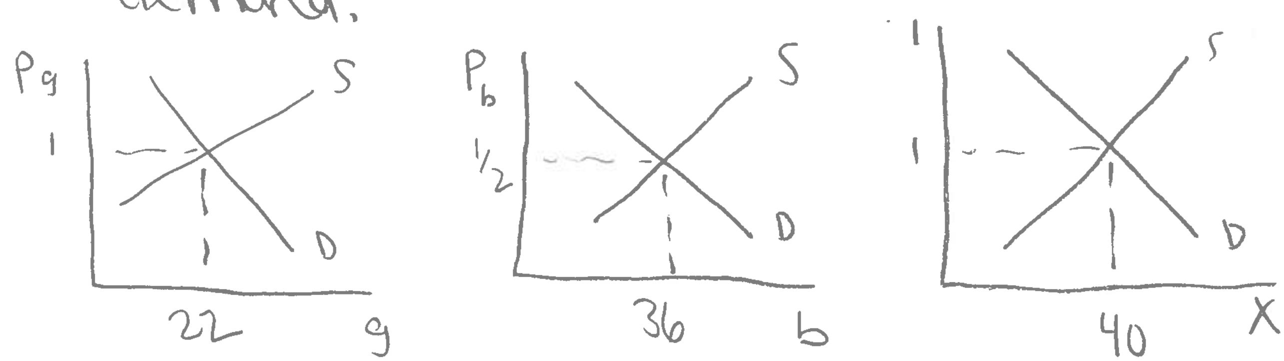
$b_A = 24$ $\qquad b_A + b_B = b = 2X_b = 36!$

$b_B = 12$

e. $X_g = 22$

$X_b = 18$

f. Yes, see part a & b. No, there is no excess demand.



FINISH

g.