HOMEWORK 1: FODE and Stability Analysis

Due: Wednesday, September 3rd, 2014
20 points

LEARNING OBJECTIVE

- In this class, a differential equation relates some function of one or more variables with its derivative of time (or shows us how a variable will change over time). It is our task to figure out the functional equations underlying the differential equations for prediction purposes.
- Determining the stability of an equilibrium point is important because it represents behavior which cannot be easily changed – good for predictions.

INSTRUCTIONS

Carefully read the questions before answering. Make sure your pages are in order and stapled together. Your work should be clear and easy to follow. When prompted, make sure you've explained your answers completely.

QUESTION

1. (20 points) As seen in class, consider a population size $x(t)$ that cannot exceed a limit, most often called a carrying capacity, K, with population growth rate $a > 0$. If the rate of change in the population is proportional to the deviation from K the differential equation governing the evolution of the population is:

$$\dot{x}(t) = a(K - x(t))$$

   a. Find the solution to this differential equation using the integrating factor method. Note, you will need to rearrange the differential equation to solve using this method (or get it into $\dot{x}(t) + px(t) = Q$ form).
   b. Solve for the steady state in this system.
   c. Determine the trajectories around the steady state (i.e. if $x > x^{ss}$ or if $x < x^{ss}$ what happens to $x(t)$)? Graph the trajectories.
   d. Use linear stability analysis to determine the stability around the steady state. Does this result match with your result and intuition from part c?