EXAM 2: Risk and Game Theory

Monday, May 4th 2015
50 points

INSTRUCTIONS

Carefully read the questions before answering. Make sure your pages are in order. Your work should be **clear and easy to follow**. When prompted, make sure you've explained your answers completely.

QUESTION

1. How many vitamins should an individual take each day? The problem is that vitamins are expensive, but people want a strong immune system, and a strong immune system requires a lot of vitamins. Suppose the average citizen preferences over wealth can be represented by a von Neumann-Morgenstern utility function, with an initial wealth of W. The probability of getting sick is given by p. The cost of vitamins is given by $C(v)$, where v is the number of vitamins. The damage done to our health if we get sick is a function of the number of vitamins we take, $D(v)$. Both $C(v)$ and $D(v)$ are twice continuously differentiable. Assume the average citizen is risk neutral.
   a. Consider a representative expected utility maximizer; set up his problem and characterize how the individual chooses the optimum number of vitamins. That is, interpret the first-order condition, which may require assumption on the above functions.

$$\max_{n}(1-p)\big(W-c(v)\big)+p\big(W-c(v)-D(v)\big)$$
$$FOC:\ -c'-pD'=0$$

OR

$$MC=MB$$

Where MC= cost per unit of n and MB=expected marginal benefits from reduced damages

$$SOC:\ -c"-pD"<0$$

   b. What assumptions will ensure the solution is a maximum?

$$D'<0, -pD''-c''<0$$

   c. How would a change in the probability of getting sick affect the number of vitamins he uses?

$$\frac{\partial v}{\partial p}=-\frac{-D'}{SOC}>0$$

d. How do your answer to a.)-b.) change if the probability is endogenous? Specifically, you can assume that the number of vitamins has a negative effect on the probability, where $p(v), p' < 0$.

$$\max_n (1 - p(v))\big(W - c(v)\big) + p(v)\big(W - c(v) - D(v)\big)$$

$$FOC: c' = -pD' - p'D$$

$$MC = MB$$

Now MB is larger because n has an impact on the probability of getting sick.

2. Undeterred by their experiences with chicken so far (as we saw during our example in class, below), James and Dean decide to increase their excitement (and the stakes) by starting their cars farther apart. This way they can keep the crowd in suspense longer, and they'll be able to accelerate to even higher speeds before they may or may not be involved in a much more serious collision. The new game table has a higher penalty for collision:

		Dean	
		Swerve	Straight
James	Swerve	0,0	-1,1
	Straight	1,-1	-10,-10

a. What are the pure-strategy Nash equilibrium of this game?

{Swerve, Straight}

{Straight, Swerve}

b. What is the mixed-strategy Nash equilibrium for this more dangerous version of chicken? What is the expected payoff to each player in this mixed-strategy equilibrium?

0p – 1(1 – p) = 1p – 10(1 – p) p = 9/10

0q – 1(1 – q) = 1q – 10(1 – q) q = 9/10

James' expected payoff = 9/10 – 10(1 – 9/10) = – 1/10

Dean's expected payoff = 9/10 – 10(1 – 9/10) = – 1/10

c. Do James and Dean play Straight more or less often than the original game shown below (Yes- you need to calculate the mixed-strategy for the game below). Discuss.

Dean

		Swerve	Straight
James	Swerve	0,0	-1,1
	Straight	1,-1	-2,-2

In the mixed-strategy Nash equilibrium in b James plays 9/10(Swerve) + 1/10(Straight), and Dean plays 9/10 (Swerve) + 1/10(Straight). Now, they each play straight ½ of the time. James and Dean play Straight more often in part c than they do in part b because the damages are higher in part b.

3. An economy has two types of jobs, Good and Bad, and two types of workers, Qualified and Unqualified. The population consists of 60% Qualified and 40% Unqualified. In a Bad job, either type of worker produces 10 units of output. In a Good job, a Qualified worker produces 100 units, and an Unqualified worker produces 0. There is enough demand for workers that for each type of job, companies must pay what they expect the appointee to produce.

   Companies must hire each worker without observing his type and pay him before knowing his actual output. But Qualified workers can signal their qualification by getting educated. For a Qualified worker, the cost of getting educated to a level n is $n^2/2$, wheras for an Unqualifed worker, it is n^2. These costs are measured in the same units as output, and n must be an integer.

   a. What is the minimum level of n that will achieve separation?

Qualified workers: $100 - 0.5n^2 > 10$, so $n^2 < 180$, or $n \leq 13$

Unqualified workers: $10 > 100 - n^2$, or $n^2 > 90$, or $n \geq 10$

With separation, the qualified get an income of 100 but pay a cost of $0.5(10^2) = 50$ for education, so they get a payoff of 100 – 50 = 50. The unqualified get a payoff of 10.

   b. Now suppose the signal is made unavailable. For example, there is not enough schooling available to achieve the separation level of n. Which kind of jobs will be filled by which kinds of worker and at what wages? Who will gain and who will lose from this change?

On a bad job, the expected output is 10. Therefore, good jobs will offer 60 and everyone will take them. Bad jobs will go unfilled. Both sides fare better in this case when the signal is unavailable.

   c. Discuss policy implications from these results.

Answers will vary, but should involve improving education availability.

4. Suppose that two identical firms are operating at the cartel solution and that each firm believes that if it adjusts its output the other firm will adjust *its* output so as to keep its market share equal to ½. What does this imply about the conjectural variation?

Constant market share means that *y*1=(*y*1 + *y*2) = 1=2, or *y*1 = *y*2. Also, the reaction function would resemble something similar to $q2(q1) = a - bq1$. Therefore, the conjectural variation is 1. We have seen that the conjectural variation that supports the cartel solution is *y*2=*y*1. In the case of identical firms, this is equal to 1. Hence, if each firm believes that the other will attempt to maintain a constant market share, the collusive outcome is stable.