1 Costs of Production

To understand this chapter, we need to understand economic or opportunity costs. These costs are the value to the best alternatvive use, which could be implicit or explicit.

$$\text{Full economic costs} = \text{implicit cost - explicit cost}$$

Other costs we'll encounter:
→ Capital costs
→ Sunk costs

Costs of production are the sum of the cost of inputs used to produce the output. These costs may be fixed or variable. In the short run, firms may have both fixed and variable costs, but in the long-run all costs are variable.. Lets first consider short-run costs and then move to the long-run.

1.1 Short-run Costs

Consider again a two input production function
if capital is constant in the short run

$$q^{sr} = f^{sr}(\overline{K}, L)$$

So in the short run the firm has both fixed costs (from the fixed amount of capital they hire) and *variable costs* (from the labor they hire). The f*ixed costs* of capital are of course related to physical capital of a firm, such as their machinery or production facility - thus we need to know the amount of fixed amount of capital the firm employs and what this costs per unit (typically a rental rate, or for example the rental rate of machinery that a firm might lease over long periods at a known cost per unit or time etc). Variable costs (in this example) only come from labor costs. To then calculate variable costs we need to know:

1. how much is produced
2. what is used to produce it (as given by the production function $q^{sr} = f^{sr}(\overline{K}, L)$ which we also know as the total product curve and tells us how much labor we need to produce output for the fixed level of capital)
3. and we need to know the wage rate.

Lets go back to our fertilizer example:

In the short run a firm produces tons of fertilizer using labor that costs $15 per hour and using the given fixed capital machinery that costs $50 per hour to rent. Production and costs are given by:

L	q^{sr}	FC	VC	C^{sr}
1	5	50	15	65
2	12	50	30	80
3	21	50	45	95
4	28	50	60	110
5	33	50	75	125
6	36	50	90	140
7	37	50	105	155

Plot q, VC and C^{sr}.

Average costs very important to firms. In the short-run, they are interested not only in average total costs AC^{sr} but also average variable costs AVC^{sr} and average fixed costs AFC^{sr} :

$$AC^{sr} = \frac{C^{sr}}{q} \;\&\; AVC^{sr} = \frac{VC}{q} \;\&\; AFC^{sr} = \frac{F}{q}$$

It may also be useful to note that in a case such as our example where labor is the only variable input in the short-run, where the wage rate is w (we assume input prices to be constant too) and L units of labor are hired, then $VC = wL$, so we can compute:

$$AVC^{sr} = \frac{VC}{q} = \frac{wL}{q} = \frac{w}{(q/L)} == \frac{w}{AP_L}$$

Marginal costs are also very important, especially as we know that firms use these to make optimal choices! Remember that marginal costs are the additional costs for an additional unit produced (or the cost of producing the next unit of output):

$$MC = \frac{\Delta C}{\Delta q} \approx \frac{dC}{dq}$$

but to produce another unit we need to have hired more labor! Thus we should really be writing this (this is an application of the Chain rule from calculus):

$$MC = \frac{\Delta VC}{\Delta q} = \frac{w\Delta L}{\Delta q} = \frac{dC}{dL}\frac{dL}{dq}$$

The additional output created by every additional unit of labor is:

$$\frac{\Delta q}{\Delta L} = MP_L$$

so we can calculate:

$$MC = \frac{w}{MP_L}$$

OK so lets put this all together:

L	q^{sr}	FC	VC	C^{sr}	AP_L	MP_L	AVC	AC	MC
1	5	50	15	65	5	5	3	13	3
2	12	50	30	80	6	7	2.50	6.67	2.14
3	21	50	45	95	7	9	2.14	4.52	1.67
4	28	50	60	110	7	7	2.14	3.93	2.14
5	33	50	75	125	6.6	5	2.27	3.79	3.00
6	36	50	90	140	6	3	2.50	3.89	5.00
7	37	50	105	155	5.3	1	2.84	4.19	15.00

Plot AP_L, MP_L, MC, AVC, AC.

Then sketch general shapes and discuss 8 general points about the shapes of cost curves.

1. \$ on the y-axis, output on the x-axis
2. C and VC are upward sloping (vertical distance is the amount of FC)

Diminishing marginal returns to labor cause the VC to rise more in proportion as output increases.

3. AVC, AC, and MC fall first then rise.
4. AC falls (with greater specialization of workers) and rises (e.g. too many workers that get in each other's ways).
5. The AC (AVC) at a particular level of output (e.g. q) is the slope of the line from the origin of the C (VC) curve to the point corresponding to q.
6. MC is the lsope of the C or VC curve (note that C and VC are parallel).
7. MC vs. AC (AVC): Note that when $MC < AC(AVC), AC(AVC)$falls; when $MC > AC(AVC), AC(AVC)$rises. $MC = AC(AVC)$ at$AC_{\min}(AVC_{\min})$.
8. $AVC_{\min}$ occurs at a lower level of output than $AC_{\min}$.

1.2 Effects of taxes on costs

Taxes shift some or all of the cost curves.

If the government collects a specific tax per unit of output, then it affects VC, TC, AVC, AC (but not FC, AFC).

If the government collects a franchise tax/business license fee, then it affects FC, AFC (but not the other costs).

1.3 Long-run Costs

There are many ways to produce a product. In the long-run (LR) a firm can adjust its employment of all inputs (i.e. of both capital and labor) to achieve the least expensive method of producing a given quantity of output or there are **NO fixed costs in the long run.** We will first run through a development of how a firm minimizes the cost of production by making input choices, and then see how long-run cost curves relate to short-run curves developed above.

1.3.1 Cost Minimization

Consider again a firm that produces its output q through a two input production function

$$q = f(K, L)$$

Firms can produce different technologically efficient combinations of inputs. It will choose the bundle that *costs the least*, i.e. the economically efficient bundle.

The firms total cost (cost functions) is:

$$C = wL + rK$$

w = wage per hour
L = hours of labor services
r = rent per hour
K = hours of machine services

In 2D: the firm's technology $q = f(K, L)$ is given by isoquants and costs $C = wL + rK$ by isocost lines that trace out the locus of all combinations of inputs that have the same level of cost γ. In 2D these are families of linear lines with equations that depend on what input is on the horizontal axis and what input on the vertical axis - for example if L is on the horizontal axis and K on the vertical axis, then the isocost line would be given by:

$$K = \frac{\overline{C}}{r} - \frac{w}{r}L$$

which is easy to plot with vertical intercept $\frac{\overline{C}}{r}$, horizontal intercept $\frac{\overline{C}}{w}$, and slope given by the **input price ratio** $\left(\frac{w}{r}\right)$ The point is that there is a "family" of isocost lines for every level of cost γ that emanates up and to the right of the origin, where higher lines have higher costs.

The firm's optimization problem is then pretty simple - for a given level of output q as described by a given isoquant, the firm wants to find the combination of inputs that allows them to produce q at least cost. This is of course given by the tangency of the isoquant with an isocost line. To find this tangency all we need to know is that the slopes of the two lines are equal! So as the slope of the isoquant is given by the $MRTS$ and the slope of an isocost line by the input price ratio $\left(\frac{w}{r}\right)$, the **cost min input choice** is given by:

$$MRTS = input\ price\ ratio$$

and as we know:
$MRTS = -\frac{\Delta K}{\Delta L} = \frac{MP_L}{MP_K}$
and
input price ratio $= \frac{w}{r}$
The the firms cost min choice is given by:

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

but just like the consumer's optimal choice - there are two variables and only one equation here - so we need some more information? What do we know? Well, we know that q is given and we know the production function $q = f(K, L)$ so to find the firm's cost min choices you need to simultaneously solve:

$$\begin{aligned} \frac{MP_L}{MP_K} &= \frac{p_l}{p_k} \\ q &= f(K, L) \end{aligned}$$

for a given q.

3 equivalent rules for choosing the optimal (cost-minimizing) bundle of inputs
1. Where lowest isocost touches IS
2. Where IS is trangent to isocost
3. Last-dollar rule: where the last dollar spent on one input gives as much extra output as the last dollar spent on any other input.

1.3.2 Factor price changes

If wage falls:

→ New isocost line will have flatter slope; intercepts change because firm's total cost falls.

→ Stay on the same IS (new point of tangency) since an input price change doesn't affect it (i.e. doesn't affect technological efficiency).

1.4 The Expansion Path

A firm's LR cost function can be found from connecting the cost minimizing input choices for each level of output - each cost minimizing choice provides the minimum cost combination of inputs to producing any level of output. In input space (L, K) for all of our above examples, we call this collection of cost min points the firm's **expansion path**, which is simply the locus of tangencies between isocosts and isoquants (for any given set of input prices). In output space, this collection of points just traces out the firm's LR total cost curve! (***Note difference between SR expansion path and LR**)

Example:

To produce its output (fertilizer) a firm combines L and K as follows, with cost per unit of L of \$15 and cost per unit (rental cost) of K of \$10 per unit:

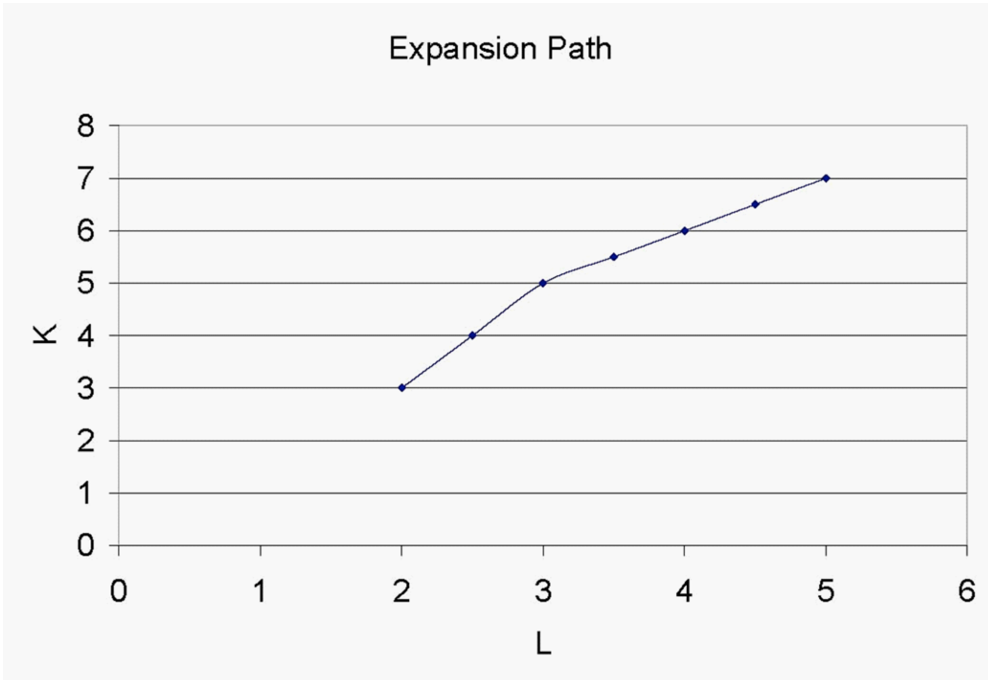
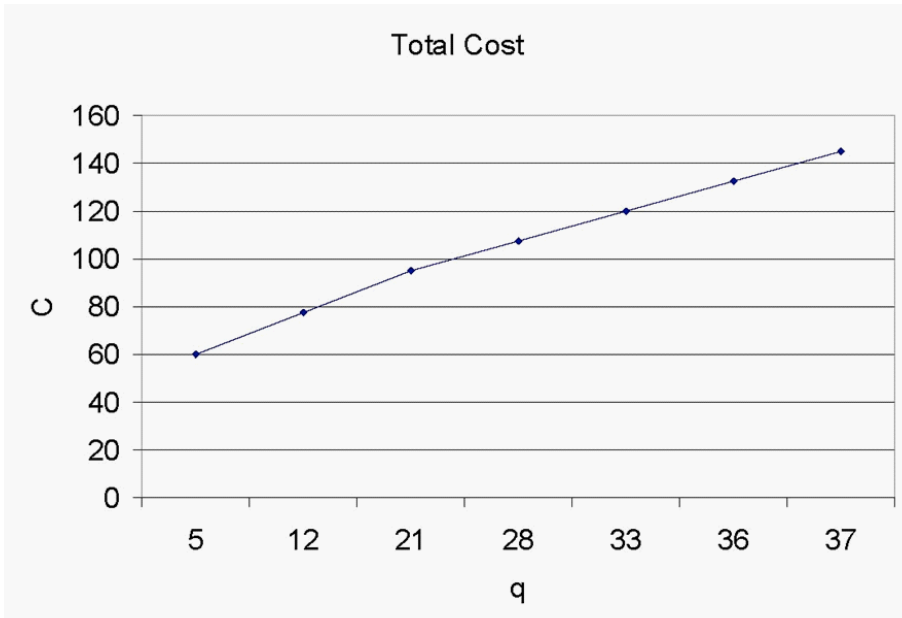


Figure 1: Expansion Path

q	K	L	rK	wL	C
5	3	2	30	30	60
12	4	2.5	40	37.50	77.50
21	5	3	50	45	95
28	5.5	3.5	55	52.50	107.50
33	6	4	60	60	120
36	6.5	4.5	65	67.50	132.50
37	7	5	70	75	145

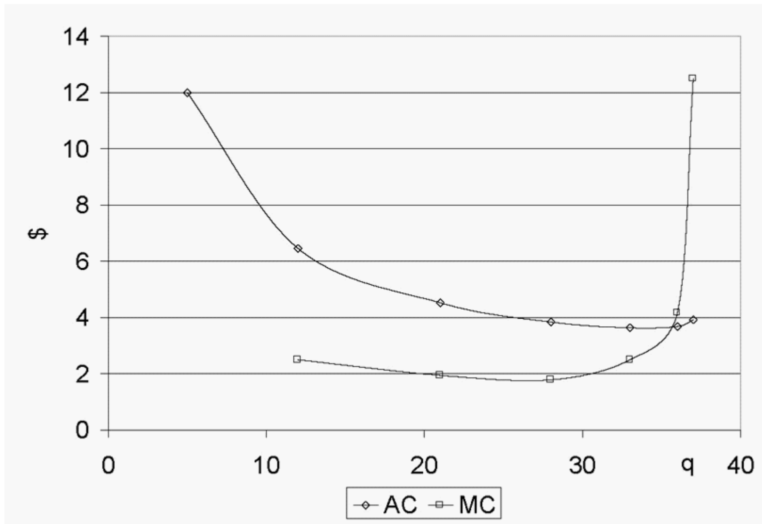


Total cost

Long-Run Cost Curves To find a firm's LR cost curves all you need is the production function, and input prices! Furthermore, if you can figure out a firm's LR cost, you can immediately figure out the firm's $AC = \frac{C}{q}$ and $MC = \frac{\Delta C}{\Delta q}$ curves (now of course being in the LR). For the above example:

q	K	L	p_{k}K	p_{l}L	C	AC	MC
5	3	2	30	30	60	12	
12	4	2.5	40	37.5	77.5	6.458333	2.5
21	5	3	50	45	95	4.52381	1.944444
28	5.5	3.5	55	52.5	107.5	3.839286	1.785714
33	6	4	60	60	120	3.636364	2.5
36	6.5	4.5	65	67.5	132.5	3.680556	4.166667
37	7	5	70	75	145	3.918919	12.5

Data



Long run cost curves

Of course what should be immediately apparent to you is that there is no AVC curve - b/c all costs in the LR are variable!

LR Costs and Returns to Scale (RTS) As we now know that costs come from the quantity to be produced, input prices and the firm's production function, then the characteristics of the production function end up as characteristics of costs! Thus as we know production can exhibit increasing, constant and decreasing returns to scale, how do these concepts affect costs?

Let's think this one through. With increasing returns to scale we said that if a firm doubles its inputs, it will more than double its output. So total costs will double, but as q goes up by more than 2, AC falls! So under increasing returns to scale, the firm's LRAC curve is decreasing! Thus an increase in production will lower AC= **Economies of scale.**

With constant returns to scale, we said that if a firm doubles its inputs, output will also double. Thus total costs will double and as output doubles, with constant returns to scale AC is constant! Thus the curve of LRAC is flat with CRS.

With decreasing returns to scale, we said that if a firm doubles its inputs, output will rise but by less than double. So total costs will double but output goes up by less than 2, so AC rises. Under DRS then the firm's LRAC curve is rising = **Diseconomies of scale**.

In general, one might expect that firm's experience IRS, then CRS, then DRS as they expand their production. This results in U-shaped AC curves, a generality we will follow.

From the SR to the LR Now we are ready to put all of this together. Lets extend our last example:

Short Run

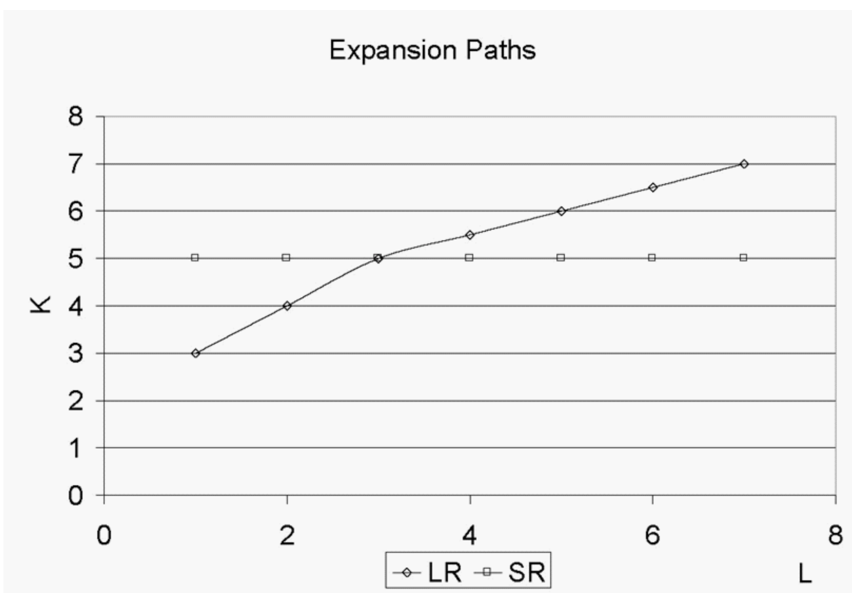
q	K	L	p_{k}K	p_{l}L	SRTC
5	5	1	50	15	65
12	5	2	50	30	80
21	5	3	50	45	95
28	5	4	50	60	110
33	5	5	50	75	125
36	5	6	50	90	140
37	5	7	50	105	155

Long Run

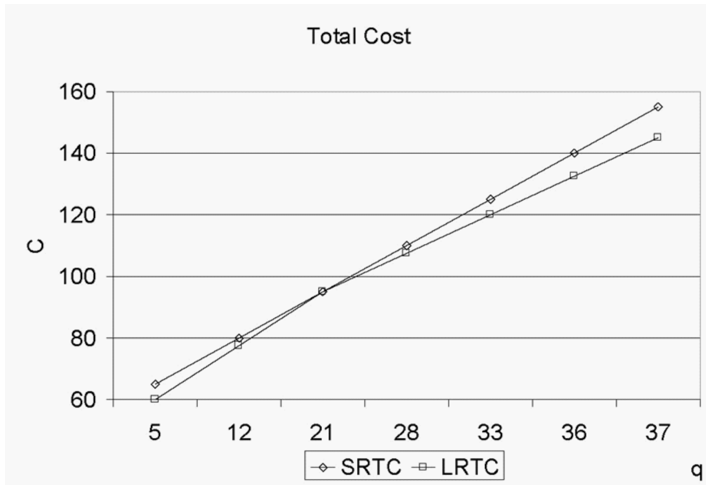
q	K	L	p_{k}K	p_{l}L	LRTC
5	3	2	30	30	60
12	4	2.5	40	37.5	77.5
21	5	3	50	45	95
28	5.5	3.5	55	52.5	107.5
33	6	4	60	60	120
36	6.5	4.5	65	67.5	132.5
37	7	5	70	75	145

Data

With graphs:



Expansion paths in SR and LR



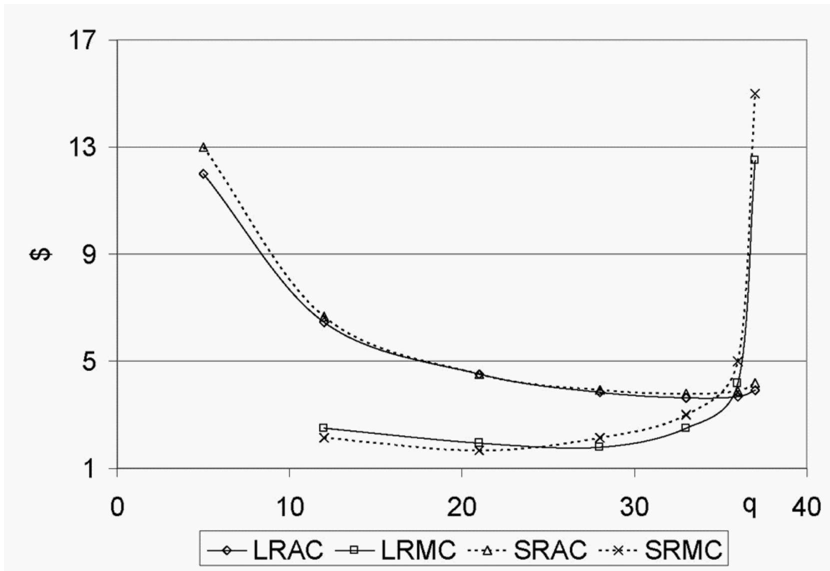
Total costs in SR and LR

But of course we might be most interested in AC and MC in SR and LR. We have a problem here - in the SR, for each fixed level of K there is a difference set of SR cost curves. It might help at this point to think of K as capacity - fixed in SR but variable in the LR. So for our data we have:

			Short Run				
q	K	L	p_{k}K	p_{l}L	SRTC	SRAC	SRMC
5	5	1	50	15	65	13	
12	5	2	50	30	80	6.666667	2.142857
21	5	3	50	45	95	4.52381	1.666667
28	5	4	50	60	110	3.928571	2.142857
33	5	5	50	75	125	3.787879	3
36	5	6	50	90	140	3.888889	5
37	5	7	50	105	155	4.189189	15

			Long Run				
q	K	L	p_{k}K	p_{l}L	LRTC	LRAC	LRMC
5	3	2	30	30	60	12	
12	4	2.5	40	37.5	77.5	6.458333	2.5
21	5	3	50	45	95	4.52381	1.944444
28	5.5	3.5	55	52.5	107.5	3.839286	1.785714
33	6	4	60	60	120	3.636364	2.5
36	6.5	4.5	65	67.5	132.5	3.680556	4.166667
37	7	5	70	75	145	3.918919	12.5

Data again



Curves in SR and LR

Of course firms operate in the short-run all the time, only occasionally do they get to make LR decisions. Regardless, as we know firms will minimize costs regardless of being in the SR or the LR, we know they will be very interested in producing at the lowest AC they can. Thus for any given level of output, in the short-run they will choose the least cost combo of inputs they can given their fixed level of capacity; but if they can lower their AC per unit in the LR they will change capacity to lower their AC.

Thus, you should expect SRTC curves to lie above the LRTC curve and touch at one point: the point where the fixed capital stock (or capacity) is optimal. When this is the case, the firm's SR and LR choices coincide. Thus the LRTC curve is the "lower boundary" of the family of SRTC curves.

Similarly with AC, to produce each unit of output the firm would like to choose the AC curve that produces that level of output at min AC. This of course requires that they can choose capital (capacity) and so is a LR choice, but of a SRAC curve. Thus SRAC curves are a family of U's, where the "lower boundary" of lowest AC for each level of output traces out the LRAC curve.

So LRAC is always below (or at) SRAC because of
→ Greater flexibility
→ Technical progress
→ Learning-by-doing - experience pays over time or over cumulative output.

1.5 Costs of producing multiple goods

So far, we've assumed a firm produces 1 good- but only for simplicity. Sometimes, cheaper to produce 2 goods (that use same inputs) jointly than separately, which is **economies of scope**.

e.g. produce both beef and leather from cattle; or tires and rubber mats from rubber.

$SC = [c(q_1, 0) + c(0, q_2) - c(q_1, q_2)]/c(q_1, q_2)$

If cheaper to produce jointly ($SC > 0$) $\rightarrow$ economies of scope
If $SC < 0 \rightarrow$ diseconomies of scope

Production possibilities frontier (PPF) combinations of output produced when input is fixed.