1 Firms & Production

We've seen demand, now we need supply.

We will see how firms organize their production efficiently and how their costs change as input prices change and the level of output (production) changes.

It will be surprising how similar the optimization problems of firms and consumers, so understanding consumer behavior will help us understand producer behavior.

1.1 Ownership of firms

→ Sole proprietorship - run by individual; full liability

→ Partnership - jointly owned; shared liability between partners

→ Corporation - owned by shareholders; limited liability

1.2 What owners want (go through and remove some items for students plus derivatives)

To maximize profits by producing as EFFICIENTLY as possible (i.e. makes the out output given the quantity of inputs used given existing technology)

→ At its core, the theory of production and costs is central to the economic management of the firm.

(For example, think of the problems that a firm like GM faces:

- how much assembly line machinery and how much labor should it use in its new plants?

- if it wants to increase production, should it hire more workers or construct new plants?

- does it make sense for one auto plant to produce different models or should each model be manufactured in a separate plant?

- what should GM expect its costs to be during the coming year and how are these costs likely to change over time, and be affected by the level of production?)

→ So the first question becomes, what do firms do?

- use FACTORS of PRODUCTION to produce final goods and services for consumers.

1.2.1 Factors of production

- land and natural resources
- labor
- capital (physical)
- intermediate goods

→ The second question is how do they do it?

- we assume firms maximize profits π
- $\pi = R - C$

where costs are economic and include the opportunity costs of production.

Thus the decision making process of a firm boils down to some thing just like we had with the consumer: when making choices they weigh the additional benefits of a choice against the additional costs of the choice and when these additionals are equated their choice is optimal: so they just apply the economic decision rule as did consumers.

Of course there are many differences between the specifics firms face in relation to consumers, and our first purpose has to be to figure out what benefits are to firms and what costs are to firms:

Benefits to firms $= R = pq$ where (to a competitive firm) p is given by the market but they choose q

Costs to firms $= C$ and these depend on what is produced q!

To apply the economic decision rule we need marginal benefits and marginal costs:

So for firms we need MR and MC:

$MR = \frac{\Delta R}{\Delta q} \approx \frac{dR}{dq}$

$MC = \frac{\Delta C}{\Delta q} \approx \frac{dC}{dq}$

So the firms optimal choice will be the q where:

$$MR = MC$$

(Example:

Small farmer owns 6 acres planted with soy beans and is needs to decide how many acres to spray with insecticide.

By spraying 1 acre, the farmer can save \$6 worth of crops.

By spraying 2 acres, the farmer can save \$11 worth of crops....

The farmer can hire a crop duster who charges \$3 an acre and this is constant per acre. The table:

Acres Sprayed	R	MR	MC	C	π
0	0			0	0
1	6	6	3	3	3
2	11	5	3	6	5
3	15	4	3	9	6
4	18	3	3	12	6
5	20	2	3	15	5
6	21	1	3	18	3

So optimal choice is to choose 4 acres (or 3).

What if the crop duster charges a flat fee of \$4 for flying out (or even the possibility of flying out):

Acres Sprayed	R	MR	MC	C	π
0	0			4	-4
1	6	6	3	7	-1
2	11	5	3	10	1
3	15	4	3	13	2
4	18	3	3	16	2
5	20	2	3	19	1
6	21	1	3	22	-1

Same optimal choice - so FIXED costs don't impact the choice, all that matters is the balance of MR and MC to apply the economic decision rule - which MAXIMIZES PROFITS!)

We know about prices etc. from markets but we need to figure out q and how it is produced and what it costs, which (as we note above) is from the firm employing inputs - so we have to get into some details in a hurry.

There is one huge difference with the basic theory of the firm and consumer theory, namely the importance of time. With consumers we assumed they could make any choice they wanted as long as they could afford it. But with firms, it seems obvious that time starts to play an important role. This is because of the fact that there are some inputs the firm can adjust at any time of their choosing, and others which take longer to adjust. We there for split out theory of the firm into two basic definitions of time: the short run (some things are fixed, they cannot change at least one input) and the long run (all inputs are variable to the firm). This concept will recur throughout our analysis so be careful that it makes sense.

First we will study the technological relationships between a firms inputs and outputs and then cover the relationship between costs and output.

1.3 Production Functions

To figure out where q comes from, we use the concept of a production function to describe how firms EFFICIENTLY combine inputs to produce output.

- by efficient here I mean that the same level of output could not be produced if any input were decreased all else constant.

Production function

- describes what is technically feasible when a firm operates efficiently
- indicates the output q that a firm produces for every specified combination of inputs
- describes the firm's technology and dictates how the firm's production responds to changes in inputs

Example:

let $q =$ quantity of barley produced

using inputs of

$L=$ labor
$K=$ capital
$D=$ land
we then write the production function as:

$$q = f(L, K, D)$$

and q then is the maximum amount of barley that can be produced with combinations of L, K, and D.

1.3.1 Production in the short run

In the short run at least some inputs to the firm are fixed. In this case, the firm then makes choices as best they can, given the fixed input. The nice thing for us is that if we have a two input production function

$$q = f(K, L)$$

and if capital is constant in the short run, then

$$q^{sr} = f^{sr}(\overline{K}, L)$$

So in the short run the firm has limited options. In this case all the firm can do to change their output (which they may want to do to max profits) is to change labor. For example, to increase q they need to hire more labor; but what will that additional labor provide in additional production? We call this the **marginal product of labor:**
$MP_L^{sr} = \frac{\Delta q^{sr}}{\Delta L} \approx \frac{dq^{sr}}{dL}$
And as capital is constant the interpretation is easy: MP_L^{sr} is the additional output from an additional unit of labor all else equal. It can be approximated by the slope of the shortrun production function with respect to (wrt) L.

The firm is also going to be interested in what, on average, is each unit of labor they have employed producing? This is the **average product of labor** and is found as:
$AP_L = \frac{q}{L}$.

Numeric Example:
In the short run a firm produces tons of fertilizer using labor according to:

L	q	MP_L	AP_L
1	5	5	5
2	12	7	6
3	21	9	7
4	28	7	7
5	33	5	6.6
6	36	3	6
7	37	1	5.3

Note with the graphs:

- The MP_L, and AP_L curves both are inverted U's due to diminishing returns, but diminishing marginal and diminishing average returns set it at different input levels.

- Total product (TP) graph has output on y-axis, labor (or whatever you assume to be the varying input) on the x-axis

- AP and MP graph (on same grid) will have AP and MP read off of the y-axis with labor on the x-axis.

- AP and MP rise first then fall.

- AP rises (with greater specialization of workers) and falls (e.g. too many workers that get in each other's way).

- The AP at a particular level of labor (e.g. L*) is the slope of the line from the origin of the TP curve to the point on the TP corresponding to L*

- MP rises (each extra worker contributes increasing output) and falls (each extra worker still contributes to output but contribution gets smaller). It falls before AP falls.

- As a result of the rise-fall MP behavior, TP curve rises while MP is positive (note that MP is the **slope** of the TP) then falls when MP becomes negative (e.g. slope of TP becomes
negative).

★ MP vs. AP: Note that when $MP > AP$, AP rises; when $MP < AP, AP$ falls. $MP = AP$ at $AP_{\max}$.

WHY? - because whenever $MP_L > AP_L$ adding additional workers causes the average product to rise:

Let $L = 5$ and $q_0 = 500$ and let $\Delta L = 1$ and let $q_1 = 630$:

$\frac{\Delta q}{\Delta L} = MP_L = \frac{630-500}{1} = \frac{130}{1} = 130$
and
$AP_L^5 = \frac{q}{L} = \frac{500}{5} = 100$
$AP_L^6 = \frac{q}{L} = \frac{630}{6} = 105$
then
add one more unit of L :
$\Delta L = 1$, $q_2 = 700$
so
$MP_L = \frac{700-630}{1} = 70$
and
$AP_L^7 = \frac{q}{L} = \frac{700}{7} = 100$
So when $MP_L < AP_L$ adding another L causes the average to fall.

We can show this graphically easily as well and note that we could repeat the entire analysis for L being fixed, K flexible, to find MP_K and AP_K and both in the long run.

Law of diminishing marginal returns/product The law of diminishing marginal returns states that as the use of an input rises (in equal increments and with all other inputs fixed) at some point the additional output produced declines. But you have to be careful to remember that this only pertains to a certain technology and not the quality of the input.

- Happens in the short-run.

1.3.2 Production in the long-run - multiple inputs

So as you can notice, with production functions where firms have more than a single input, we are going to have trouble with a graphical description. So we again use the concept of level curves (just as we did in consumer theory), expect now the levels are called **isoquants** and they depict certain constant levels of output that can be produced using alternative combinations of inputs.

For a **two input** production function

$$q = f(K, L)$$

the function is a 3D dome which we take horizontal slices of to find isoquants, where each isoquant reflects a constant level of output (for combinations of inputs).

Just like indifference curves *the slope of the isoquant turns out to be very, very important and show how the firm can substitute one input for another while keeping production constant.*

We call the slope of an isoquant the:**Marginal rate of technical substitution (**$MRTS$) and

$$MRTS = -\tfrac{\Delta K}{\Delta L}$$

for a certain level of output q and with K on the vertical axis and L on the horizontal axis.

And there are important relationships between marginal products and the marginal rate of technical substitution. (Mathematically this can be found easily be taking what is known as the total differential of q but lets figure it out intuitively):

We know that on an isoquant q is constant. If we move from one point on the isoquant to another, capital and labor change, but output remains the same.

Start at some point on an isoquant.

Add some capital (ΔK) and reduce some labor (ΔL) with quantities that keep q constant.

From the additional capital we get a little bit of additional output, which is equal to the marginal product of capital multiplied by the additional capital we use:

$$\text{extra production from change in } K = (MP_K)\,\Delta K$$

The corresponding decrease in labor (to keep q constant) can be found from multiplying the marginal product of labor by the reduced labor we use:

$$\text{reduced production from change in } L = \ (MP_L)\,\Delta L$$

and we know the total change in output must be zero:

$$\Delta q = 0 = (MP_K)\,\Delta K + (MP_L)\,\Delta L$$

So we can find the $MRTS$ as:

$$MRTS = -\frac{\Delta K}{\Delta L} = \frac{MP_L}{MP_K}$$

Thus the marginal rate of technical substitution between two inputs is equal to the ratio of the marginal products of the inputs.

The beauty of the $MRTS$ us that it tells us how a firm can substitute amongst alternative inputs to maintain a certain level of output, and so tells us about **changes in the input mix** and how a firm **moves along an isoquant**.

Thus the $MRTS$ measures the **rate** at which the technology of the firm will allow substitution of one input for the other to keep output constant (and is thus the slope of the isoquant).

Example: commercial fishermen, q=tons of fish, $x_1 = deckhands$, $x_2 = nets$, x_1 on the horizontal axis and x_2 on the vertical

q	x_1	x_2	Δx_1	Δx_2	$MTRS = -\frac{\Delta x_2}{\Delta x_1}$
8	10	15			
8	15	10	5	-5	1
8	20	6	5	-4	4/5
8	25	5	5	-1	1/5

So, the $MRTS$ lets us understand how a firm substitutes one input for another to keep output constant - but, what does the firm have to do to produce MORE output (i.e. increase all inputs!)?

Remember that in the SR some inputs are fixed and not immediateley adjustable, while in the LR all inputs can be varied - so a LR question that we need to answer is **what a firm has to do to increase output.** To answer this question we need to understand **returns to scale.**

1.3.3 Returns to Scale

How much does output change if the firm increases all its inputs proportionately?

Constant returns to scale (CRS) - if increase inputs by x%, output increases by x %

$$f(2L, 2K) = 2f(L, K)$$

Increasing returns to scale (IRS) - if increase inputs by a x%, output increase by more than x %

$$f(2L, 2K) > 2f(L, K)$$

Decreasing returns to scale (DRS) - if increase inputs by a x%, output increases by less than x%

$$f(2L, 2K) < 2f(L, K)$$

Cobb-Douglas production function - one of the more widely estimated production functions

$$q = AL^{\alpha}K^{\beta}$$

where A, α, β are all (+) constants

Varying RTS

For some firms, there is IRS for low output levels; CRS for moderate output levels and DRS for high output levels.

1.3.4 Technical progress/innovation

- advance in knowledge that allows more output to be produced with the same level of inputs

1. Neutral technical change - can produce more output using same ratio of inpputs.
2. Non-neutral technical change - changes the proportion of input use (e.g. could become *labor-saving* or *capital-saving*)

1.3.5 Organizational change

- better management or organization that allows more output to be produced with the same level of inputs.

(If we scale up the amount of all inputs by some constant factor - what happens to output?

If we double inputs and this results in a doubling of output - **constant returns to scale.**

If we double inputs and this results in more than a doubling of output - **increasing returns to scale.**

If we double inputs and this results in less than a doubling of output - **decreasing returns to scale.)**