1 Finding the Derivative of a Function: A Beginner's Guide

Rule for taking the derivative of a function (a function is a mathematical expression that shows how the value of one variable, e.g. f below- depends on another (or more) variable/s). In our applications, "marginal" concepts (i.e. marginal utility or marginal product) are derivatives of the "total" functions (i.e. total utility function or total production function). Graphically, the "marginal" concepts are also the slopes of the "total" curves.

Assume x and y are variables that affect the value of f; c is a constant; n and m are exponents

Example 1:

$$f = cx^n + cy^n$$

derivative of f with respect to x $\rightarrow \frac{df}{dx} = \mathbf{cnx}^{n-1}$
(keep the constant as is; 2nd term becomes 0 since it doesn't have x in it)

derivative of f with respect to y $\rightarrow \frac{df}{dy} = \mathbf{cny}^{n-1}$
(keep the constant as is; 1st term becomes 0 since it doesn't have y in it)

So if for instance, your production function is $q = 5L + 3K^4$,then:

Marginal product of labor=$\text{MP}_L = \frac{dq}{dL} = 5$
*(note that $\text{L}^0 = 1$, i.e. anything raised to 0 is 1)

Marginal product of capital=$\text{MP}_K = \frac{dq}{dK} = 12K^3$

Example 2:

$$f = cx^n y^m$$

derivative of f with respect to x $\rightarrow \frac{df}{dx} = (cy^m)nx^{n-1}$
(keep the cy^m as is since it doesn't have x in it and focus on the x term; you can't make $cy^m = 0$ as we did in Example 1 since it's part of the big term)

derivative of f with respect to y $\rightarrow \frac{df}{dy} = (cx^n)my^{m-1}$
(keep the cx^n as is since it doesn't have y in it and focus on the y term; you can't make $cx^n = 0$ as we did in Example 1 since it's part of the big term)

So if for instance, your utility function is $U = 2X^3Y^2$, then:
Marginal utility of $X = MU_X = \frac{dU}{dX} = (Y^2)6X^2$
Marginal utility of $Y = MUY = \frac{dU}{dY} = (2X^3)2Y = 4X^3Y$